香港中文大學（深圳）
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Introduction to Computer Science: Programming Methodology

Lecture 11
Tree

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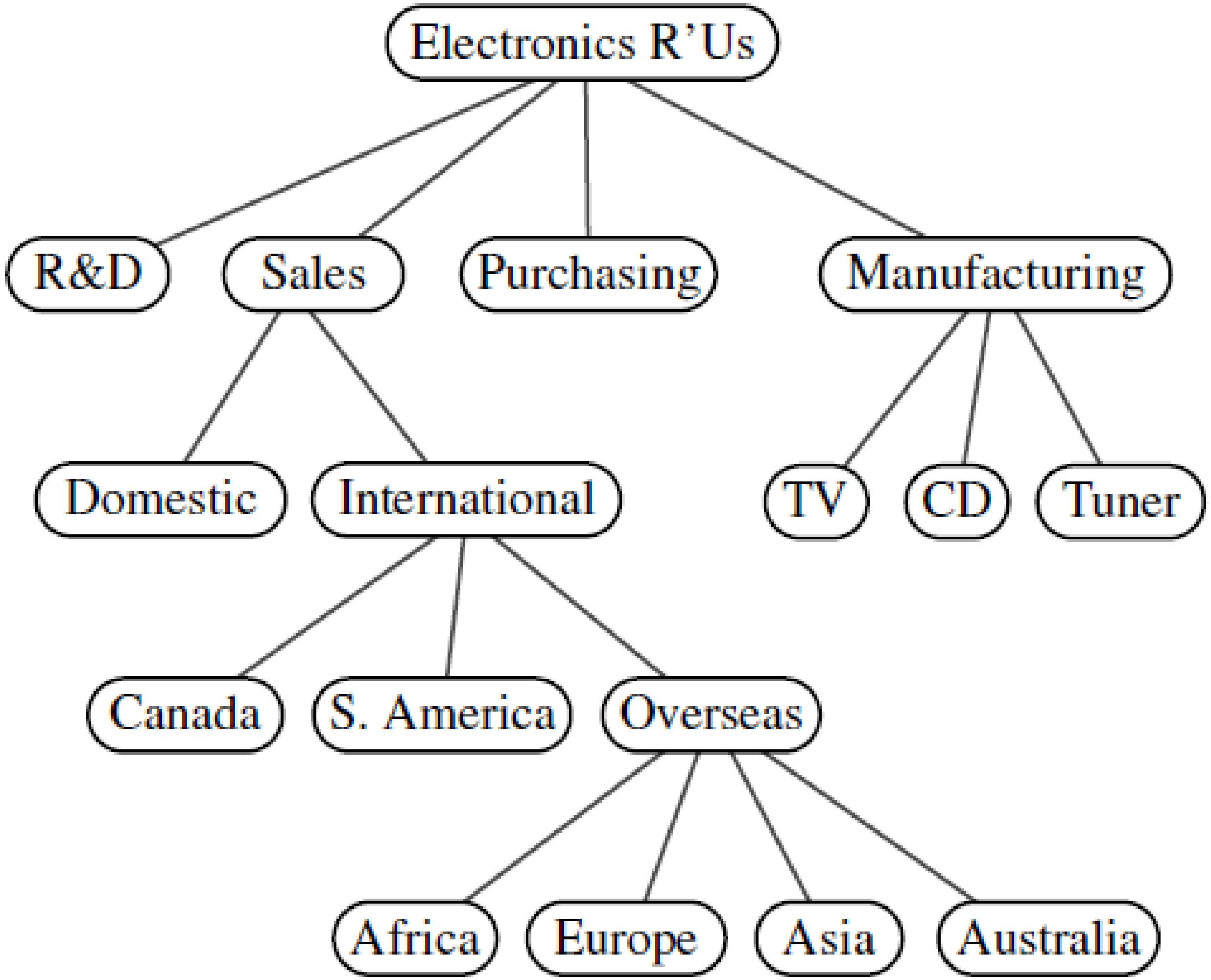
Tree

- A tree is a data structure that stores elements hierarchically

- With the exception of the top element, each element in a tree has a parent element and zero or more children elements

- We typically call the top element the root of the tree, but it is drawn as the highest element

Example: The organization of a company



Formal definition of a tree

- Formally, we define a tree T as a set of nodes storing elements such that the nodes have a parent-child relationship that satisfies the following properties:

  ✓ If T is nonempty, it has a special node, called the root of T, that has no parent.

  ✓ Each node v of T different from the root has a unique parent node w; every node with parent w is a child of w.

Edge and path

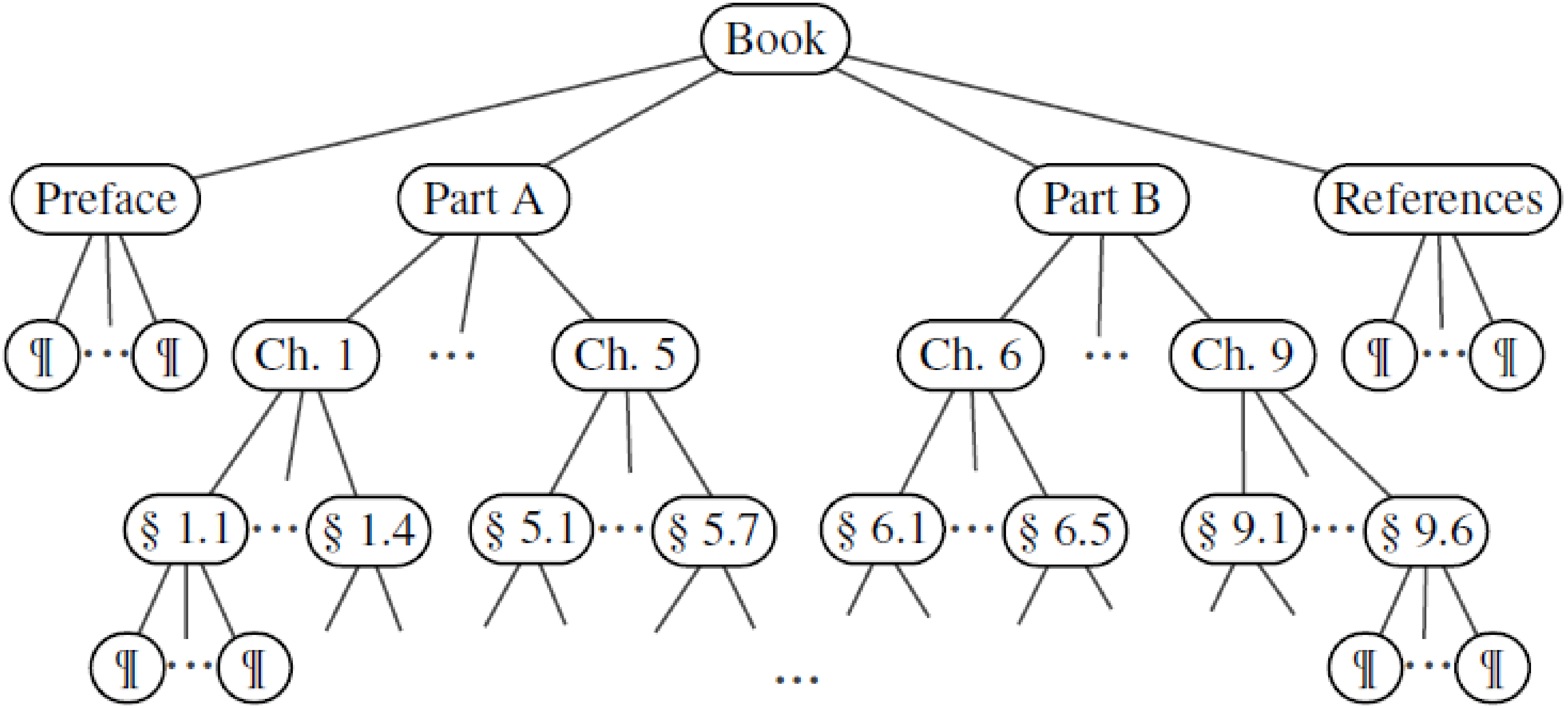
- An edge of tree T is a pair of nodes (u,v) such that u is the parent of v, or vice versa
- A path of T is a sequence of nodes such that any two consecutive nodes in the sequence form an edge
- The depth of a node v is the length of the path connecting root node and v

Internal and leaf nodes

- A node is called a leaf node if it has no child
- If a node has at least one child, it is an internal node

Ordered tree

- A tree is ordered if there is a meaningful linear order among the children of each node; such an order is usually visualized by arranging siblings from left to right, according to their order



Binary tree

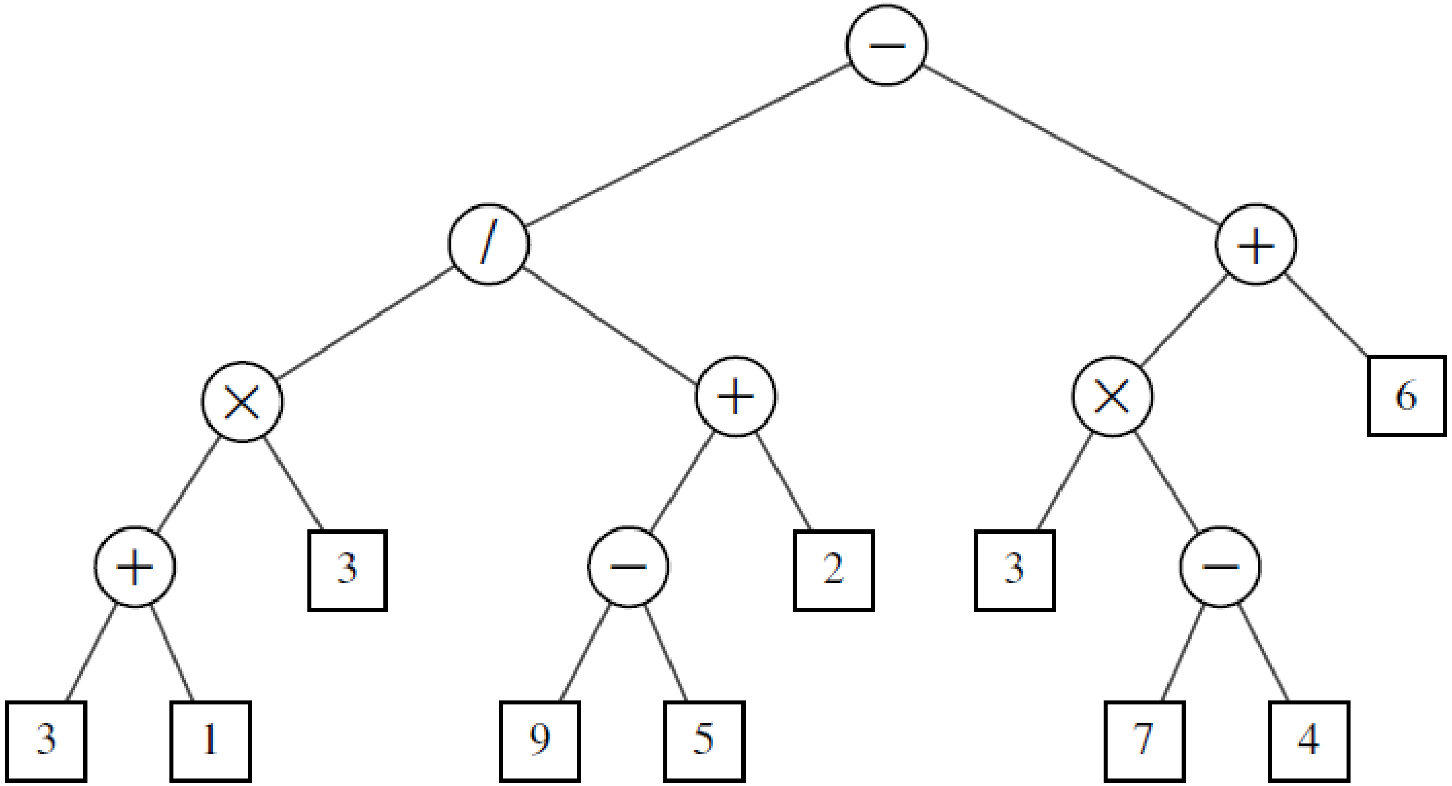
- A binary tree is an ordered tree with the following properties:
  1. Every node has at most two children
  2. Each child node is labeled as being either a left child or a right child
  3. A left child precedes a right child in the order of children of a node

The subtree rooted at a left or right child of an internal node v is called a left subtree or right subtree, respectively, of v

A binary tree is proper if each node has either zero or two children. Some people also refer to such trees as being full binary trees

Example: Represent an expression with binary tree

- An arithmetic expression can be represented by a binary tree whose leaves are associated with variables or constants, and whose internal nodes are associated with one of the operators +, −, ×, and /



Binary tree class

- We define a tree class based on a class called Node; an element is stored as a node

- Each node contains three references, one pointing to the parent node, two pointing to the child nodes

Implementing the binary tree

```
class Node:

    def __init__(self,element, parent = None,\
        left = None, right = None):
        self.element = element
        self.parent = parent
        self.left = left
        self.right = right

class LBTree:

    def __init__(self):
        self.root = None
        self.size = 0

    def __len__(self):
        return self.size
```

```
    def find_root(self):
        return self.root

    def parent(self,p):
        return p.parent

    def left(self,p):
        return p.left

    def right(self,p):
        return p.right

    def num_child(self,p):
        count = 0
        if p.left is not None:
            count+=1
        if p.right is not None:
            count+=1
        return count
```

Implementing the binary tree

```
def add_root(self,e):
    if self.root is not None:
        print('Root already exists.')
        return None
    self.size = 1
    self.root = Node(e)
    return self.root

def add_left(self,p,e):
    if p.left is not None:
        print('Left child already exists.')
        return None
    self.size+=1
    p.left = Node(e,p)
    return p.left
```

```
def add_right(self,p,e):
    if p.right is not None:
        print('Right child already exists.')
        return None
    self.size+=1
    p.right = Node(e,p)
    return p.right

def replace(self,p,e):
    old = p.element
    p.element = e
    return old

def delete(self,p):
    if p.parent.left is p:
        p.parent.left = None
    if p.parent.right is p:
        p.parent.right = None
    return p.element
```

Example: Use the binary tree class

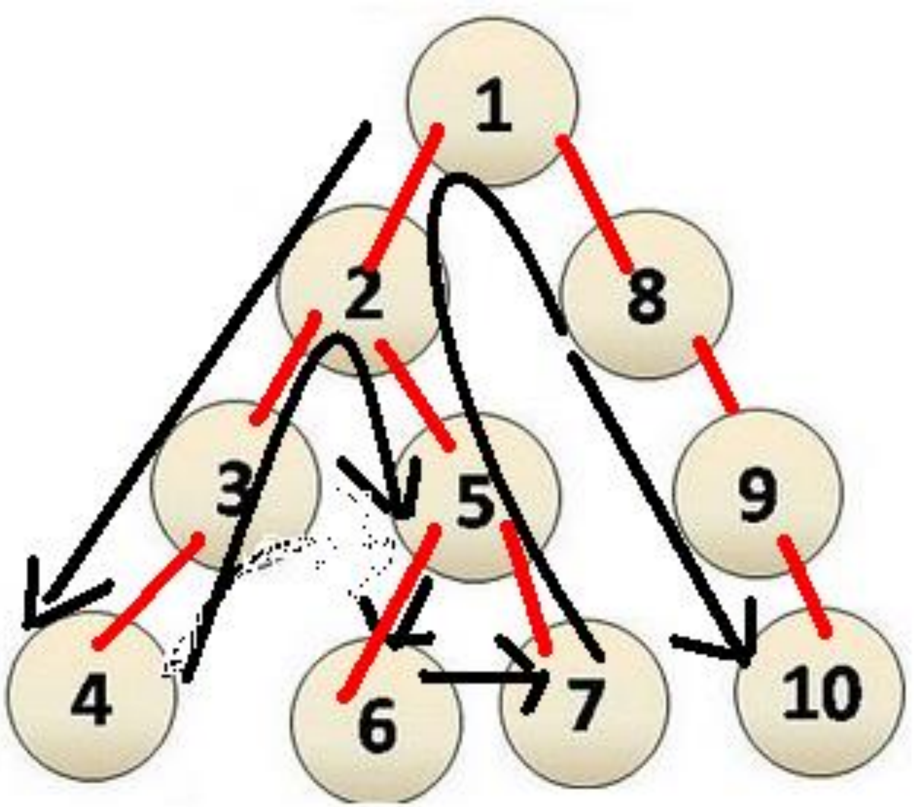
```
def main():
    t = LBTree()
    t.add_root(10)
    t.add_left(t.root, 20)
    t.add_right(t.root, 30)
    t.add_left(t.root.left, 40)
    t.add_right(t.root.left, 50)
    t.add_left(t.root.right, 60)
    t.add_right(t.root.left.left, 70)

    print(t.root.element)
    print(t.root.left.element)
    print(t.root.right.element)
    print(t.root.left.right.element)
```

```
>>> main()
10
20
30
50
```

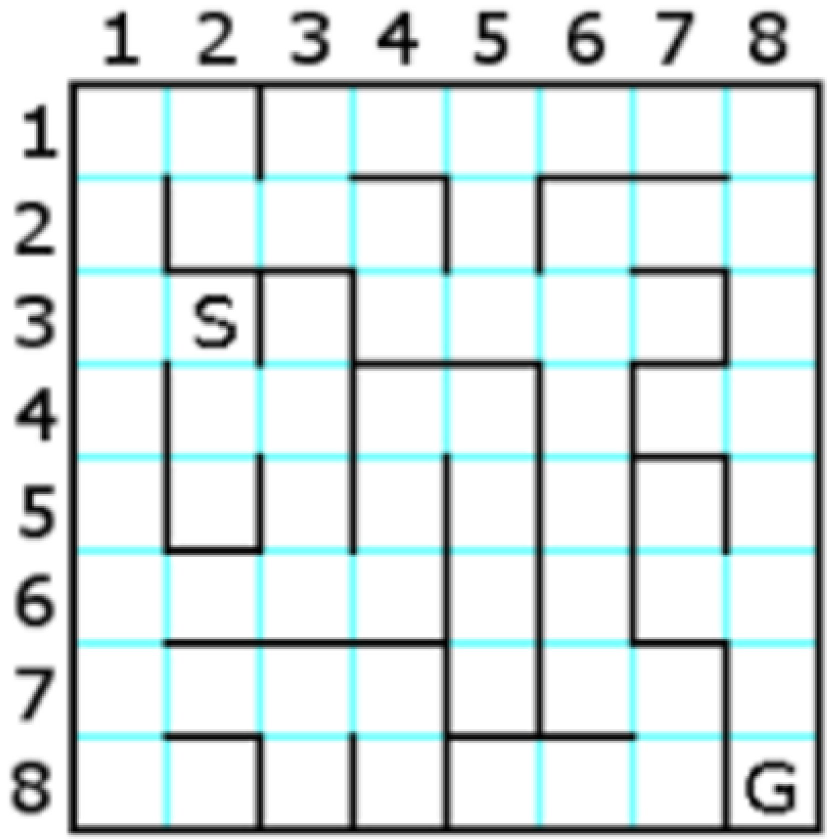
Depth first search over a tree

- Depth-first search (DFS) is a fundamental algorithm for traversing or searching tree data structures

- One starts at the root and explores as deep as possible along each branch before backtracking



Example: search a path in a maze



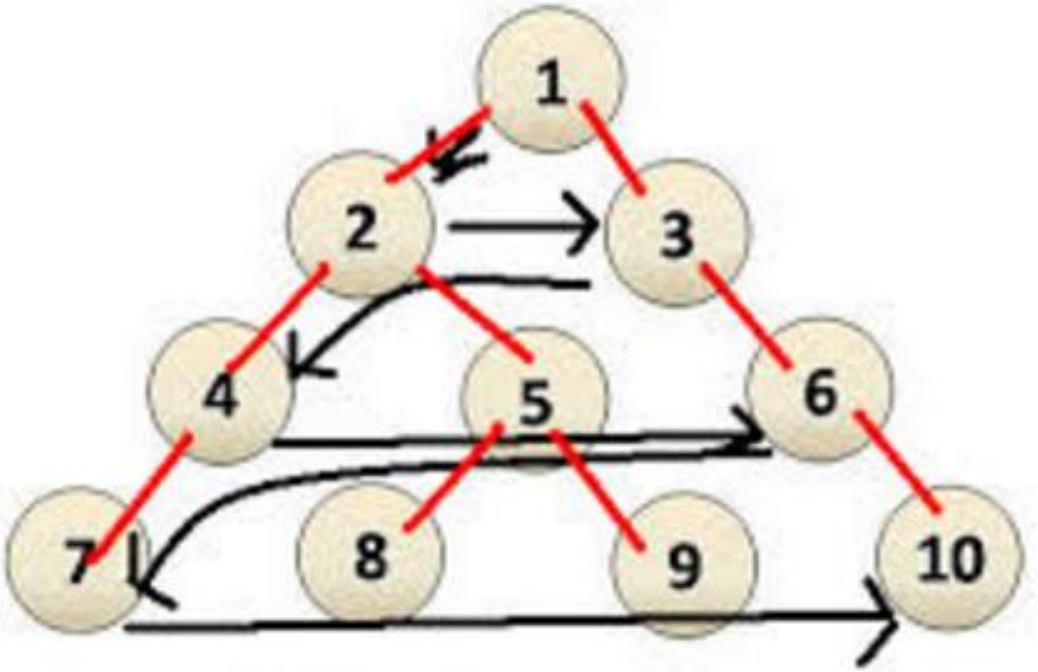
The code of DFS over a binary tree

```
def DFSearch(t):
    if t:
        print(t.element)
    if (t.left is None) and (t.right is None):
        return
    else:
        if t.left is not None:
            DFSearch(t.left)
        if t.right is not None:
            DFSearch(t.right)
```

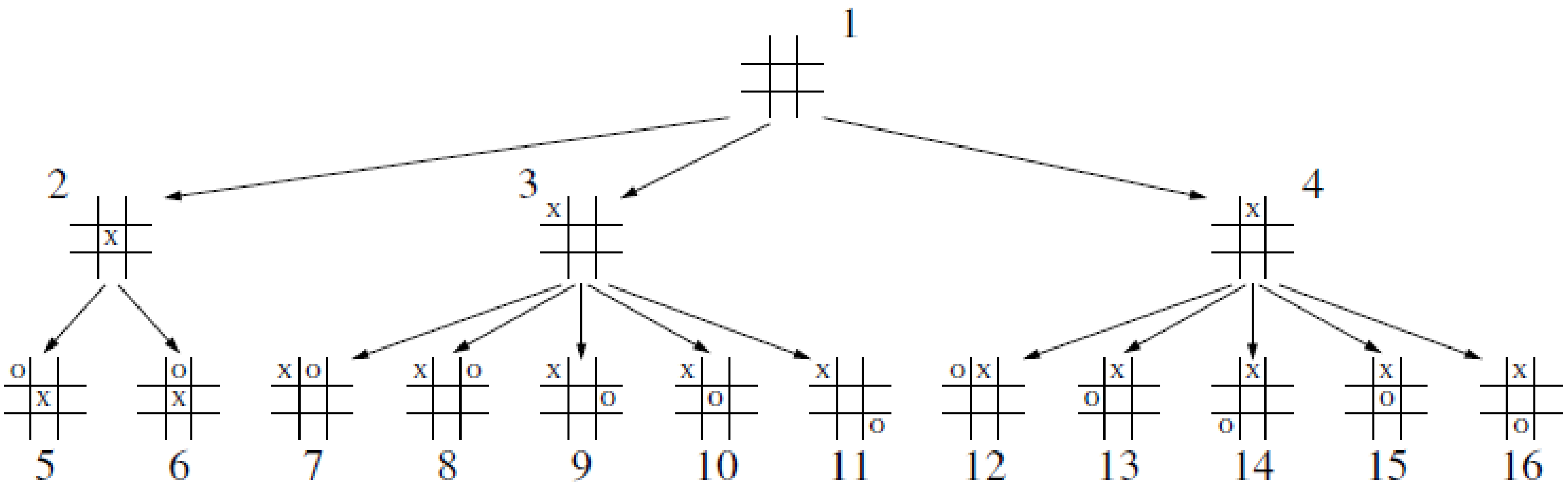
Breadth first search over a tree

- Breadth-first search (BFS) is another very important algorithm for traversing or searching tree data structures

- Starts at the root and we visit all the positions at depth d before we visit the positions at depth d +1



Example: finding the best move in a game



The code of BFS over a binary tree

```
def BFSearch(t):

    q = ListQueue()
    q.enqueue(t)

    while q.is_empty() is False:
        cNode = q.dequeue()
        if cNode.left is not None:
            q.enqueue(cNode.left)
        if cNode.right is not None:
            q.enqueue(cNode.right)
        print(cNode.element)
```